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Generalized Form of p-Bounded Variation of Sequences of Fuzzy Real Numbers

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Abstract: Classical mathematics deals with only two conclusions: true and false. But fuzzy logic is a multiple-valued logic in which the truth values of variables might be any real number between 0 and 1. L. A. Zadeh developed the idea of fuzzy logic in 1965 to investigate the haziness and lack of concentration in information found in mathematics. The notion of the fuzzy set has been successfully applied in studying the different classes of sequence spaces. In recent years, many researchers have replaced these mathematical structures of real or complex numbers with fuzzy numbers and interval numbers and have investigated many results. This study aims to analyze the sequence space $bV^F_p(X)$ for $1\leq p<\infty$ of p- bounded variation of fuzzy real numbers and it is extended to the p- bounded variation of the difference sequence space $bV^F_p(\Delta_m X)$ of fuzzy real numbers. The proposed study will be based on a dry lab review. It will be based on existing theories that are already proven and established. On the promise of the existing theories, we will study some new results with their different properties. To study the different properties, we will introduce a new metric on $bV^F_p(\Delta_m X)$. Moreover, we shall explore some of the inclusion relations with respect to p and q.

Keywords: Fuzzy Real Numbers, Fuzzy Set, Fuzzy Sequence, Difference Sequence of Fuzzy Real Numbers

1. Introduction

So far a bulk number of works have been done in the mathematical structures constructed with real or complex numbers. Recently, many research has been performed by replacing these mathematical structures with fuzzy numbers and investigating many results. Before the introduction of fuzzy logic and fuzzy set, mathematics was limited to only two conclusions and those are true and false (denoted by 1 and 0). The traditional view holds that science should aim for certainty in all of its expressions and that uncertainty is unscientific [16]. But fuzzy logic deals with such problems which have no clear answer i.e., vague and unfocused on the information. Thus fuzzy logic is the method of thinking that looks like human thought. Also, it is an approach to a computing-based degree of truth than the true or false (1 or 0). The theory of the fuzzy set and its operation on was firstly presented by American mathematician Zadeh [17] in 1965. Since then numerous researchers have studied various parts of its concept and application. The notion of the fuzzy set has successfully been applied in studying different fields of mathematics. A large number of researchers have used the fuzzy set and fuzzy real numbers in different classes of sequence spaces. Matloka [10] examined boundedness and convergent sequences of fuzzy numbers and demonstrated that all convergent sequences of fuzzy numbers are bounded. Talo. and Bassar F.[13] showed that the space $bv_p(F)$ includes the space $l_p(F)$ and that the spaces $bv_p(F)$ and $l_p(F)$ are isomorphic for $1 \leq p \leq \infty$. Also in 2009, The sequence spaces of fuzzy numbers were introduced and explored by Rifat C. and et. al. [12] using the difference operator Δ^m and an Orlicz function, and several of their properties, such as completeness, solidity, and symmetry were studied. The idea of the fuzzy set has been successfully applied in studying the difference sequence space of fuzzy real numbers by researchers.

In 2010, Baruah A. and Tripathy B. C [2] proposed necessary and sufficient criteria for the Nörlund and Riesz methods for transforming convergent sequences of fuzzy numbers into limit-preserving convergent sequences of fuzzy numbers. In 1981, Kizmez [9] introduced the notation and concept of difference sequence space of real or complex numbers $\mathcal{L}_\infty(\Delta), \mathcal{C}(\Delta)$, and $\mathcal{C}_o(\Delta)$ as follows:

$$l_\infty(\Delta) = \{ X = X_k : \Delta X \in l_\infty\}$$

$$C(\Delta) = \{ X = X_k : \Delta X \in C\}$$

$$C_o(\Delta) = \{ X = X_k : \Delta X \in C_o\}$$

where $\Delta X = X_k - X_{k+1}$.

This idea was generalized by Et. and Colka [7] in 1995, and Tripathy and Esi [15] in 2006 presented the notion of the difference operator $\Delta_m \mathcal{F}$ to study the topological properties of the sequence spaces $\mathcal{L}_\infty(\Delta_m), \mathcal{C}(\Delta_m)$ and $\mathcal{C}_o(\Delta_m)$. Also, in 2009, Baruch and Tripathy [1] introduced the difference operators Δ_m ($m \geq 0$) for positive integer m to study different properties of difference sequence spaces $\mathcal{L}_\infty^f(\Delta m), \mathcal{C}^f$, and $\mathcal{C}_0^f(\Delta m)$ of fuzzy real numbers. In 2016, Et, M., Savas, and Altnok [6] introduced and examined a few classes of sequences of fuzzy numbers and looked at some of their properties, including completeness, solidity, symmetry, and convergence free. Tripathy and Das [14] studied the sequence spaces $\mathcal{C}^f(M)$, $\mathcal{C}_0^f(M)$ and $\mathcal{L}^f(M)$ of fuzzy real numbers with the new fuzzy metric. In 2018 Das P. C. [5] used the notation $bV_p^f(\mathcal{F})$ for the fuzzy norm to study different properties such as completeness, solidness, symmetricity, and convergence free. Jalal [8] used ideal convergence and the modulus function to introduce some multi-ordered difference sequence space for fuzzy real numbers. In 1989, Nanda [11] introduced the space $\mathcal{L}_p^f$ of fuzzy numbers defined by

$$\mathcal{L}_p^f = \{ \mathcal{F} = (\mathcal{F}_k) : \textstyle\sum_n [\, \bar{d}\,(\mathcal{F}_n, 0)]^p < \infty \}$$

Tripathy and Das [14] modified that space l_p studied by Nanda [11] and introduced the new class of fuzzy real numbers $bV_p^f(\Delta_m), for\ 1 \leq p < \infty$.

2. Class of p-Bounded Variation of Sequence of Fuzzy Real Numbers

Let G be the set of all bounded intervals H= [a, b] on the real line ℝ. Then for any $H, E \in G$ with $H = [a_1, b_1]$ and $E = [a_2, b_2]$ then H ≤ E if $a_2 \leq a_1$ and $b_1 \leq b_2$. Define a relation ρ on G by

$$\rho(H, E) = max\{ |a_2 - a_1|, |b_2 - b_1|\}$$

Then clearly, ρ defines a metric in G, and obviously the metric space (G, ρ) is complete.

Definition 2.1[3]

A fuzzy set $\mathcal{F}$ is a fuzzy real number i.e. a mapping $\mathcal{F}: \mathbb{R} \to I = [\,0, 1]$ relating each $t \in \mathbb{R}$ with $\mathcal{F}$ (*t*), called membership value.

The fuzzy number X is

i. Normal if there is $t \in \mathbb{R}$ with $\mathcal{F}$ (t) = 1;
ii. Convex if for $t, s \in \mathbb{R}$ and $0 \leq \theta \leq 1$, $\mathcal{F}$ (θ t + (1 – θ) s) ≥ min { $\mathcal{F}$ (t), $\mathcal{F}$ (s) };
iii. $\mathcal{F}$ is upper semi-continuous if for $\varepsilon > 0, \mathcal{F}^{-1}([0, a + \varepsilon))$, $\forall a \in I$ is open in the usual topology of ℝ.

The α- level set on fuzzy set $\mathcal{F}$ is denoted by $\mathcal{F}^\alpha$ and defined by $\mathcal{F}^\alpha = \{ t \in \mathbb{R}: \mathcal{F}(t) \geq \alpha \}$..

Support of a fuzzy number is the set of fuzzy numbers having a membership value greater than zero.

Assume that $\mathbb{R}(I)$ is the collection of all upper semi-continuous fuzzy numbers with compact support. In other words, $\mathcal{F} \in \mathbb{R}(I)$ then for α, with $0 \leq \alpha \leq 1$,

$$\mathcal{F}^\alpha = \begin{cases} t: \mathcal{F}(t) \geq \alpha\ for\ \alpha \in (0, 1] \\ t: \mathcal{F}(t) > \alpha\ for\ \alpha = 0 \end{cases}$$

The definition of the addition and scalar multiplication on $\mathbb{R}(I)$ is,

$[\mathcal{F} + \mathcal{H}]^\alpha = \mathcal{F}^\alpha + \mathcal{H}^\alpha$ and $(a\mathcal{F})^\alpha = a\, \mathcal{F}^\alpha, \forall\, \alpha \in [\,0, 1]$

Consider a relation $\bar{\rho}$: $\mathbb{R}(I)$ x $\mathbb{R}(I) \to \mathbb{R}$ by the relation

$$\bar{\rho}(\mathcal{F}, \mathcal{H}) = \sup_{0 \leq \alpha \leq 1} d(\mathcal{F}^\alpha, \mathcal{H}^\alpha)$$

Then clearly $\bar{\rho}$ is a metric on $\mathbb{R}(I)$ and the metric space $(\mathbb{R}(I), \bar{\rho})$ is complete.

Then for any $\mathcal{F}, \mathcal{H} \in \mathbb{R}(I), \mathcal{F} \leq \mathcal{H}$ if and only if $[\mathcal{F}^\alpha] \leq [\mathcal{H}^\alpha]$ for $\alpha \in [\,0, 1]$ and

$$\mathcal{F}^\alpha = [\, x_1^\alpha, x_2^\alpha\,] \text{ and } \mathcal{H}^\alpha = [\, y_1^\alpha, y_2^\alpha\,].$$

Definition 2.2[4]

A sequence $\mathcal{F}$ = ($\mathcal{F}_k$) of fuzzy numbers is a function $\mathcal{F}: \mathbb{N} \to \mathbb{R}(I)$, where $\mathbb{N} = \{ 0, 1, 2, 3, 4, \dots, \dots, \dots \}$. The fuzzy number $\mathcal{F}_k$ is the k^th^ (k ∈ ℕ) value of the function and is the k^th^ term of the sequence.

If there are fuzzy numbers M and m such that $m \leq \mathcal{F}_k \leq M$ then a sequence of fuzzy numbers $\mathcal{F}$ = ($\mathcal{F}_k$) is said to be bounded.

A fuzzy sequence $\mathcal{F}$ = ($\mathcal{F}_k$) is said to be convergent to $l \in \mathbb{R}(I)$ if $\forall\, \varepsilon > 0, \exists n_o \in \mathbb{N}: \bar{\rho}(\mathcal{F}_k, l) < \varepsilon\ \forall\, k \geq n_o$ and we write $\lim_{k \to \infty} \mathcal{F}_k = l$

It is said to be Cauchy sequence $\forall \varepsilon > 0, \exists n_o \in \mathbb{N}: \forall\, m, n \geq n_o \Longrightarrow \bar{\rho}\, (\mathcal{F}_m, \mathcal{F}_n) < \varepsilon$

Now we discuss difference sequence space and then the difference sequence space of fuzzy real and its generalized form.

Kizmaz [9] first proposed the idea of the difference sequence space for a crisp set.

Definition 2.3: Let $\omega(F)$ stand for the collection of all fuzzy sequences. For $\Delta\mathcal{F} = \mathcal{F}_k - \mathcal{F}_{k+1}$, k= 1, 2, 3,…,…,…,

Let us define the following difference sequence spaces $\mathcal{L}_\infty(\mathcal{F}\Delta), \mathcal{C}(\mathcal{F}\Delta), \mathcal{C}_o(\mathcal{F}\Delta)$ of fuzzy real numbers as follows:

$$\mathcal{L}_\infty^F(\Delta\mathcal{F}) = \{\mathcal{F}_k \in \omega(F): \Delta\mathcal{F} \in \mathcal{L}_\infty^f\}$$

$$C^F(\Delta\mathcal{F}) = \{\mathcal{F}_k \in \omega(F): \Delta\mathcal{F} \in C^f\}$$

$$C_o^F(\Delta\mathcal{F}) = \{\mathcal{F}_k \in \omega(F): \Delta\mathcal{F} \in C_o^f\}$$

$$\mathcal{L}_p^F(\Delta\mathcal{F}) = \{\mathcal{F}_k \in \omega(F): \Delta\mathcal{F} \in \mathcal{L}_p^f\}$$

where,

$\mathcal{L}_\infty^f(\Delta\mathcal{F})$ = set of all bounded sequences of fuzzy numbers,

$\mathcal{C}^f(\Delta\mathcal{F})$ = set of all convergent sequences of fuzzy numbers,

$\mathcal{C}_o^f(\Delta\mathcal{F})$ = set of all null sequences of fuzzy numbers.

The class of sequences $\mathcal{L}_p^f$ $for\ 1 \le p < \infty$ of fuzzy numbers was introduced and studied as follows:

$$\mathcal{L}_p^f = \{\mathcal{F} = (\mathcal{F}_k) \in \omega(F): \sum_{k=1}^{\infty}\{\bar{\rho}(\mathcal{F}_k, 0)\}^p < \infty\}$$

Later Tripathy and Das [14] introduced the class of P-bounded variation of difference sequence space of fuzzy real numbers denoted by $bV_p{}^f$ for $1 \le p < \infty$ as follows:

$bV_p{}^f = \{\mathcal{F} = \mathcal{F}_k \in \omega(F): \sum_{k=1}^{\infty}\{\bar{\rho}(\Delta\mathcal{F}_k, 0)\}^p < \infty\}$ with $\Delta\mathcal{F}_k = \mathcal{F}_k - \mathcal{F}_{k+1}\ \forall\ k \in N$, and with the metric $\xi(\mathcal{F}, \mathcal{H}) = \bar{\rho}(\mathcal{F}_1, \mathcal{H}_1) + [\sum_{k=1}^{\infty}\{\bar{\rho}(\Delta\mathcal{F}_k, \Delta\mathcal{H}_k)\}^p]^{1/p}$ for $\mathcal{F} = \mathcal{F}_k\ and\ \mathcal{H} = \mathcal{H}_k \in bV_p^f$ showed that the class $bV_p{}^f\ 1 \le p < \infty$ is a complete metric space.

Later Burch and Tripathy B. C [1, 5, 6] generalized the difference sequence space of fuzzy real numbers denoted by $\mathcal{L}_\infty^f(\Delta_m), \mathcal{L}_p^f(\Delta_m), \mathcal{C}^f(\Delta_m)\ \&\ \mathcal{C}_O^f(\Delta_m)$ and defined as follows:

$$Z(\Delta_m) = \{\mathcal{F} = \mathcal{F}_k \in \omega(F): \Delta_m\mathcal{F}_k \in Z\}$$

for Z = $\mathcal{L}_\infty^f, \mathcal{L}_p^f, \mathcal{C}^f\ \&\ \mathcal{C}_O^f$ and $\Delta_m\mathcal{F}_k = \mathcal{F}_k - \mathcal{F}_{k+m}$ for all k ∈ N and showed that these sequence spaces are complete with metric

$$\xi(\mathcal{F}, \mathcal{H}) = \sum_{k=1}^{m}\bar{\rho}(\mathcal{F}_k, \mathcal{H}_k) + \sup_k \bar{\rho}(\Delta_m\mathcal{F}_k, \Delta_m\mathcal{H}_k)$$

Here, we discussed the class of generalized differenced sequence space of fuzzy real numbers with p- bounded variations. denoted by $bV_p^f(\Delta_m\mathcal{F})$ and defined by

$$bV_p{}^F(\Delta_m\mathcal{F}) = \{\mathcal{F} = (\mathcal{F}_k) \in \omega(F): \sum_{k=1}^{\infty}\{\bar{\rho}(\Delta_m\mathcal{F}_k, \bar{0})\}^p < \infty\}$$

for $1 \le p < \infty$, where $\Delta_m\mathcal{F}_k = \mathcal{F}_k - \mathcal{F}_{k+m}$.

3. Main Results

Here, we will look at some results that define the topological structure of a new class of p-bounded variation of generalized difference sequence spaces $bV_p{}^f(\Delta_m\mathcal{F})$ for $1 \le p < \infty$ of fuzzy real numbers.

Theorem 3.1

The class of sequence of p-bounded variation of generalized sequence of fuzzy real numbers $bV_p{}^F(\Delta_m X_k)$ defined above is complete metric space with the metric defined by

$$\bar{d}_p(\mathcal{F}, \mathcal{H}) = [\sum_{k=1}^{\infty}\{\bar{\rho}(\Delta_m\mathcal{F}_k, \Delta_m\mathcal{H}_k)\}^p]^{1/p}, 1 \le p < \infty$$

where, $\mathcal{F} = \mathcal{F}_k$, $\mathcal{H} = \mathcal{H}_k \in bV_p^f(\Delta_m\mathcal{F}_k)$ and $\Delta_m\mathcal{F}_k = \mathcal{F}_k - \mathcal{F}_{k+m}$

Proof: Let $\{\mathcal{F}^i\}$ be a Cauchy sequence in $bV_p{}^F(\Delta_m\mathcal{F}_k)$, where

$\mathcal{F}^i = (\mathcal{F}^i{}_k) = (\mathcal{F}^i{}_1, \mathcal{F}^i{}_2, \mathcal{F}^i{}_3, \mathcal{F}^i{}_4, \ldots\ldots\ldots.)$ then for all $\varepsilon > 0$ there is $n_o \in$ N such that

$$\bar{d}_p(\mathcal{F}^i, \mathcal{F}^j) = [\sum_{k=1}^{\infty}\{\bar{\rho}(\Delta_m\mathcal{F}^i{}_k, \Delta_m\mathcal{F}^j{}_k)\}^p]^{1/p} < \varepsilon \quad (1)$$

$$\Rightarrow \sum_{k=1}^{\infty}\bar{\rho}(\Delta_m\mathcal{F}^i{}_k, \Delta_m\mathcal{F}^j{}_k) < \varepsilon$$

This shows that $\{\Delta_m\mathcal{F}^i{}_k\}$ is a Cauchy sequence in ℝ(I)f or all k ∈ N. Since ℝ(I) is complete, the sequence$\{\Delta_m\mathcal{F}^i{}_k\}$ is a convergent sequence in ℝ(I) and suppose that

$$\lim_{i\to\infty}\Delta_m\mathcal{F}^i{}_k = \Delta_m\mathcal{F}_k$$

$$i.e\ \lim_{i\to\infty}(\mathcal{F}^i{}_k - \mathcal{F}^i{}_{k+1}) = \mathcal{F}_k - \mathcal{F}_{k+m}$$

i. e $\lim_{i\to\infty}\mathcal{F}^i{}_k = \mathcal{F}_k$ for all k ∈ N.

Thus, $\lim_{i\to\infty}\mathcal{F}^i = \mathcal{F}$ where $\mathcal{F} = \mathcal{F}_k$ for all k ∈ N.

To complete the proof, we need to show that $\mathcal{F} \in bV_p{}^f(\Delta_m\mathcal{F}_k)$.

For this we have from (1)

$$\bar{d}_p(\mathcal{F}^i, \mathcal{F}^j) = [\sum_{k=1}^{\infty}\{\bar{\rho}(\Delta_m\mathcal{F}^i{}_k, \Delta_m\mathcal{F}^j{}_k)\}^p]^{1/p} < \varepsilon$$

Now,

$$[\sum_{k=1}^{\infty}\{\bar{\rho}(\Delta_m\mathcal{F}_k, \bar{0})\}^p]^{1/p} = \bar{d}_p(\mathcal{F}, \bar{0})$$

$$\le \bar{d}_p(\mathcal{F}, \mathcal{F}^j) + \bar{d}_p(\mathcal{F}^i, \bar{0})$$

$$< \varepsilon' + \bar{d}_p(\mathcal{F}^i, \bar{0}) < \varepsilon$$

Hence, $[\sum_{k=1}^{\infty}\{\bar{\rho}(\Delta_m\mathcal{F}_k, \bar{0})\}^p]^{1/p} < \varepsilon$ for $1 \le p < \infty$.

$\Rightarrow \mathcal{F} \in bV_p{}^f(\Delta_m\mathcal{F}_k)$. Hence $bV_p^f(\Delta_m\mathcal{F}_k)$ is a complete metric space.

Theorem 3.2: For $1 < p < \infty$, the relation $\mathcal{L}_p^f(\mathcal{F}) \subseteq bV_p^f(\mathcal{F})$

Proof: Let $\mathcal{F} = (\mathcal{F}_k) \in \mathcal{L}_p^f$ then $\sum_{k=1}^{\infty}\{\bar{d}(\mathcal{F}_k, \bar{0})\}^p < \infty\}$. Now, we show that $\sum_{k=1}^{\infty}\{\bar{\rho}(\Delta_m\mathcal{F}_k, \bar{0})\}^p < \infty$.

For,

$$[\bar{d}(\Delta_m\mathcal{F}_k, \bar{0})]^p = \bar{[}(\mathcal{F}_k - \mathcal{F}_{k+m}, \bar{0})]^p \le [\bar{\rho}(\mathcal{F}_k, \bar{0}) + \bar{\rho}(\mathcal{F}_{k+m}, \bar{0})]^p$$

$$\le 2^p\ \{[\bar{\rho}(\mathcal{F}_k, \bar{0})]^p, [\bar{\rho}(\mathcal{F}_{k+m}, \bar{0})]^p\}$$

$$\le 2^p\ \{[\bar{\rho}(\mathcal{F}_k, \bar{0})]^p + [\bar{\rho}(\mathcal{F}_{k+m}, \bar{0})]^p\} < \infty$$

Thus $[\bar{\rho}(\Delta_m\mathcal{F}_k, \bar{0})]^p < \infty$. Hence $\mathcal{F} = (\mathcal{F}_k) \in bV_p{}^f$. So that $\mathcal{L}_p^f(\mathcal{F}) \subseteq bV_p{}^f(\mathcal{F})$.

Theorem 3.3: For $1 \le q < p < \infty$ then $bV_q{}^f(\mathcal{F}) \subseteq bV_p^f(\mathcal{F})$.

Let $\mathcal{F} = \mathcal{F}_k \in bV_q^f(\mathcal{F})$ then $\bar{\rho}(\Delta_m\mathcal{F}_k, \bar{0})]^q < \infty$. Here $\Delta_m\mathcal{F}_k \to \bar{0}$ as $\to \infty$, so ∃ positive integer n_o such that $\bar{\rho}(\Delta_m\mathcal{F}_k, \bar{0}) \le 1$ for k $> n_o$

Now,

$$\sum_{k=1}^{\infty}\{\,\bar{\rho}(\Delta_m\,\mathcal{F}_k,\bar{0})\}^p=\sum_{k=1}^{n-1}\{\,\bar{\rho}(\Delta_m\,\mathcal{F}_k,\bar{0})\}^p+\sum_{k=n}^{\infty}\{\,\bar{\rho}(\Delta_m\,\mathcal{F}_k,\bar{0})\}^p \qquad (2)$$

Since, $\sum_{k=n}^{\infty}\{\,\bar{\rho}(\Delta_m\,\mathcal{F}_k,\bar{0})\}^p \le \sum_{k=n}^{\infty}\{\,\bar{\rho}(\Delta_m\,\mathcal{F}_k,\bar{0})\}^q < \infty$, then form (2)

$\sum_{k=1}^{\infty}\{\,\bar{\rho}(\Delta_m\,\mathcal{F}_k,\bar{0})\}^p$ is a finite sum and hence $\mathcal{F}=\mathcal{F}_{\mathrm{k}} \in bV_p{}^f(\mathcal{F})$.

Thus, $bV_q{}^f(\mathcal{F}) \subseteq bV_p{}^f(\mathcal{F})$.

4. Conclusion

Several mathematicians have researched the topic of difference sequence space of fuzzy real numbers. Here, we have shown that the generalized form of the class of sequence space $bV_q{}^F$ of fuzzy real numbers is complete metric space. Also we have shown the relation $l_p^F(X) \subseteq bV_p{}^F(\mathrm{X})$ and for $1 \le p < \infty$ $bV_q{}^F(\mathrm{X}) \subseteq bV_p{}^F(\mathrm{X})$ $1 \le \mathrm{q} < p < \infty$.

References

[1] A. Baruah, and B. C. Tripathy, New type of difference sequence spaces of fuzzy real numbers. *Mathematical Modeling and Analysis*. 14 (2009) 391-397.

[2] B. Achyutananda and T. Binod Chandra, N¨orlund and Riesz mean of sequences of fuzzy real numbers. *Applied Mathematics Letters.* 23 (2010) 651- 655.

[3] C. Basudev, and T. Binod Chandra, On fuzzy real-valued $l^F{}_p$ sequences. *In Proc. International Conf. 8th Joint Con. Inf. Sci. (10th International Conf. on Fuzzy Theory and Technology*. (2005) 184-190.

[4] N. R. Das, and C. Ajanta, Boundedness of fuzzy real-valued sequences. *Bull. Cal. Math. Soc.* 90 (1998) 35-44.

[5] D. Paritosh Chandra, Fuzzy normed linear sequence space $bV^F{}_p$. *Proyecciones* (*Antofagasta, on line*). 37 (2018) 389-403.

[6] E. Mikail., S. Ekrem. and A. Hifsi, On some difference sequence spaces of fuzzy numbers. *Soft Computer* 20 (2016) 4395-4401.

[7] E. Mikail, and R. Çolak, On some generalized difference sequence spaces. *Soochow Journal of Mathematics*. 21 (1995) 377-386.

[8] J. Tanweer, A note on multiordered fuzzy difference sequence spaces. *Filomat*. 32 (2018) 2867-2874.

[9] H. Kizmaz, On certain sequence spaces. *Canadian Mathematical Bulletin*. 24 (1981) 169-176.

[10] M. Matloka, Sequence of fuzzy numbers. *Busefal.* 28 (1986) 28-37.

[11] S. Nanda, (1989), On sequences of fuzzy numbers. *Fuzzy Sets and System*. 33, 123-126.

[12] C.. Rifat, A. Hssi and E. Mikail., Generalized difference sequences of fuzzy numbers. *Chaos, Solution, and Fractions*. 40 (2009) 1109-1117.

[13] Ö. Talo, F. Bassar, On the space $bV^F{}_p$ of sequences of p-bounded variation of fuzzy numbers. *Acta. Math. Sin.-English Ser*. 24 (2008) 1205-1212.

[14] T. Binod Chandra and D. Paritosh Chandra, On the sequence of fuzzy number sequence $bV^F{}_p$. *Songklanakarin J. Sci. Technol*. 41 (2019) 11–14.

[15] T. Binod Chandra, & A. Esi, A new type of difference sequence spaces. *Int. J. Sci. Technol*. *1* (2006) 11-14.

[16] B. Yuan, & K. George, Fuzzy sets and fuzzy logic: theory and applications. *PHI New Delhi*. (1995) 443-455.

[17] L. A. Zadeh, Is there a need for fuzzy logic?. *Information Sciences*. *178* (2008) 2751–2779.